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Title: The Discrete Ordinates (S_N) Method and Lie Symmetries

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The Discrete Ordinates (S_N) Method and Lie Symmetries

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X Theoretical Design

Nuclear Threat Assessment

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Summer Fun

Student Name (Group e.g. NEN-1)

- Educational Background
  - BS Temple University, 2014
  - PhD Michigan State University, Ongoing

- X Theoretical Design
  - Nuclear Threat Assessment
  - Joe Schmidt

- Research
  - Building a 1-D discrete ordinates code and learning about Lie symmetries
  - Studying the role of cross-shell excitations in $^{70}\mathrm{Ni}$

Research Overview and Motivation

Q: Verification and validation are required to understand if a simulation is working, but how do we probe the assumptions underlying the simulation?

Research Overview and Motivation

- A: One way is to check how the assumptions affect the symmetries of the governing differential equations.

- Sophus Lie developed symmetry methods for handling differential equations in the 1870s.

1-D Neutron Transport in Slab Geometry

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\sigma(x)\psi(x,\mu)=\sum_{l=0}^{L}(2l+1)P_l(\mu)\sigma_l(x)\phi_l(x)+s(x,\mu)$$

- Flux being absorbed in the medium
- Flux scattering into direction μ_n
- External sources and fission

This equation is hard to solve. So let's just solve it for specific angles μ.

Discrete ordinates (S_N) method

We choose a specific set of ordinates μ_n for $n = 1,2, \ldots, N$. This is the

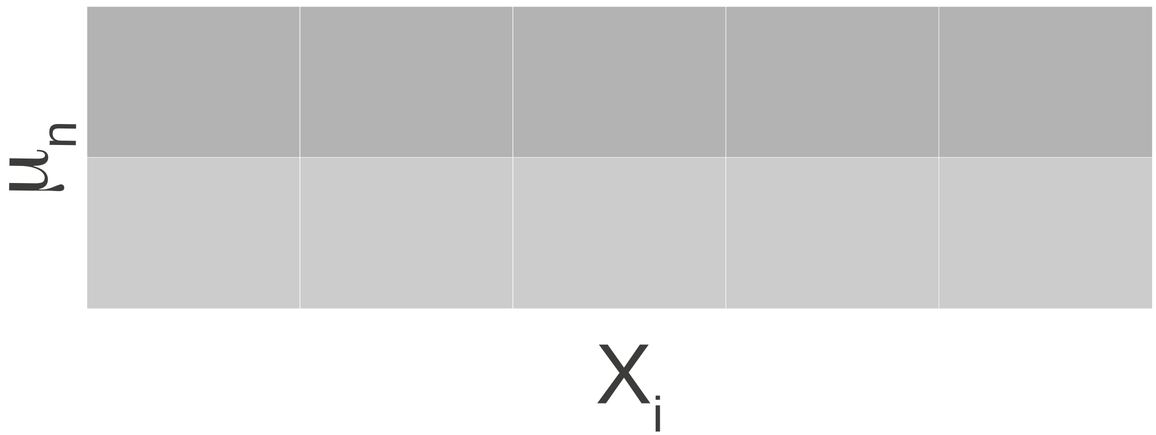
$$\mu_n \frac{d\psi_n(x)}{dx} + \sigma(x)\psi_n(x) = \sum_{l=0}^{L} (2l+1) P_l(\mu_n)\sigma_l(x)\phi_l(x) + s(x,\mu_n)$$

The set of ordinates is chosen to accurately integrate the scalar flux:

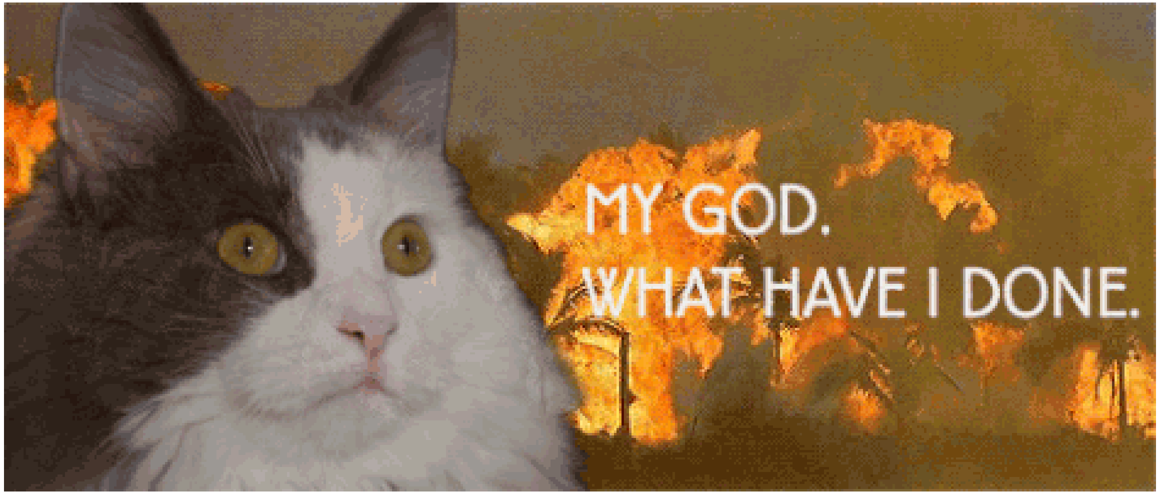
$$\phi(x) = \frac{1}{2}\int_{-1}^{1} d\mu\ \psi(x,\mu) \Rightarrow \phi(x) = \frac{1}{2}\sum_{n=1}^{N} w_n \psi_n(x)$$

Discrete ordinates (S_N) method

We then discretize our spatial variable.



We can relate the flux at each grid point with neighboring grid points.



- Learned about the discrete ordinates method, and implemented a 1-D slab geometry neutron transport solver in python using this method.

- Started learning about Lie symmetries, though no results yet.